POSITION TRACKING FOR UNDERACTUATED RIGID BODIES ON $SE(3)$

Paulo Tabuada * **Pedro Lima** *

* *Instituto de Sistemas e Robótica*
Instituto Superior Técnico
Av. Rovisco Pais, 1
1049-001 Lisboa - PORTUGAL
Phone: (351) 21 841 8270 Fax: (351) 21 841 8291
{tabuada,pal}@isr.ist.utl.pt

Abstract: The problem of tracking a desired trajectory is of fundamental importance in real applications where some system is required to follow a pre-planned or pre-specified trajectory. For underactuated systems this problem is not always solvable since the desired trajectory may not belong to the set of feasible trajectories for the given system. However real life applications often only require tracking of some of the variables, the most common example being a unicycle type robot following a preassigned 2D path. In this paper we study the problem of position tracking for underactuated rigid bodies on $SE(3)$.

Keywords: Trajectory Tracking, Nonlinear Control, Nonholonomic Systems, Underactuated Systems, Differential Geometry.

1. INTRODUCTION

Tracking a desired trajectory is a frequent problem is control and robotics, where a pre-planned path representing the accomplishment of certain goals must be enforced. This pre-specified path may represent an optimal solution for the problem, a required maneuver to be executed such as docking of a vehicle, or the outcome of some higher-level controller.

For fully actuated systems this problem is now well understood and solutions are proposed in standard textbooks on nonlinear control (Isidori, 1996) and (Nijmeijer and van der Schaft, 1990). On the other hand tracking for underactuated systems is a challenging problem from the theoretical point of view since not all trajectories are feasible by the system, and the results developed for fully actuated systems fail to apply. From the practical point of view this problem is also of great importance since the development of systems with less actuators allows for reductions in the cost of the overall system and in full actuated systems represents a valuable safeguard regarding malfunctioning of some of the available actuators.

In this article we will address a special case of this problem where the system in only required to track some of the state variables, more specifically we will consider underactuated rigid bodies on the special euclidean group $SE(3)$ where it is only required that the system tracks a reference position in three dimensional space. The importance of this problem comes from the fact that often a mission is only specified in terms of a desired position trajectory and no orientation information is available.

Traditional approaches to this problem involve linearization about the reference trajectory and methods from linear control theory resulting in

[1] The first author was supported by Fundação para a Ciência e Tecnologia under grant PRAXIS XXI/BD/18149/98.

a global gain-scheduled control law (Kaminer *et al.*, 1998) or linear time-varying control (Walsh *et al.*, 1994). Other approaches include adaptive and feedback linearization schemes (Fossen, 1994) or using constant forward speed, thereby reducing the problem to control the attitude of the rigid body towards the reference trajectory (Encarnação *et al.*, 2000). This approach was originally introduced in (Samson, 1992; Micaelli and Samson, 1993) and since then more advanced techniques have also been applied to planar robots such as partial feedback linearization and dynamic feedback linearization, (d'Abdrea Nolvel *et al.*, 1995) (Thuillot *et al.*, 1996). A survey of the various methods of control and trajectory tracking for mobile robots is given in (de Wit *et al.*, 1997) and for ocean vehicles in (Fossen, 1994).
Contrary to the described approaches, in this paper we will address the problem from a coordinate-free perspective, therefore allowing a simpler and more general understanding and presentation of the results often obscured by a particular choice of coordinates. This approach makes use of several techniques from differential geometry and has been strongly influenced by work on tracking with similar approaches such as (Bullo and Murray, 1999). A good introduction to nonholonomic systems in the context of Riemannian manifolds is given in (Bloch and Crouch, 1995).

2. MATHEMATICAL PRELIMINARIES

We shall assume that the reader is familiar with several differential geometric concepts at the level of (Boothby, 1975).

2.1 $SE(3)$, *left invariant metrics and kinematic connections.*

In this paper we will consider the left-invariant kinematic model of an underactuated rigid body in $SE(3)$ given by:

$$\frac{d}{dt}g = g.(X_1u_1 + X_2u_2 + X_3u_3 + X_4u_4) \qquad (1)$$

where $g \in SE(3)$ and X_1, X_2, X_3 and X_4 are the basis vectors of the lie algebra $\mathfrak{se}(3)$ representing the direction of motion along roll, pitch, yaw and forward translational velocity, respectively. Note that the system is underactuated since motion along the remaining basis vectors of $\mathfrak{se}(3)$ is not possible. Instead of writing elements of $\mathfrak{se}(3)$ in matrix form we will adopt the following simpler representation:

$$\{(\omega_1,\omega_2,\omega_3),(v_1,v_2,v_3)\} \leftrightarrow \begin{bmatrix} 0 & -\omega_3 & \omega_2 & v_1 \\ \omega_3 & 0 & -\omega_1 & v_2 \\ -\omega_2 & \omega_1 & 0 & v_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2)$$

which allows us to represent a left invariant metric on $SE(3)$ in the following form:

$$\Theta = \begin{bmatrix} \alpha\mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \beta\mathbf{I}_{3\times3} \end{bmatrix} \qquad (3)$$

with α and β positive scalars. For a discussion on the possible metrics on $SE(3)$ and its relation with the kinematic connection we defer the reader to (Zefran *et al.*, 1999) and the references therein. We shall also need the kinematics connection compatible with the previously given left-invariant metric and whose non zero Christofell symbols we reproduce here for completeness:

$$\begin{aligned} \Gamma^3_{12} &= \Gamma^2_{31} = \Gamma^1_{23} = \frac{1}{2} \\ \Gamma^3_{21} &= \Gamma^2_{13} = \Gamma^1_{32} = -\frac{1}{2} \\ \Gamma^6_{15} &= \Gamma^4_{26} = \Gamma^5_{34} = 1 \\ \Gamma^6_{24} &= \Gamma^4_{35} = \Gamma^5_{16} = -1 \end{aligned} \qquad (4)$$

2.2 *Error functions*

We shall define error functions on $\mathbb{R}^n$ for the sake of generality, although we are only interested in tracking trajectories in $\mathbb{R}^3$. For a definition of error functions on abstract manifolds the reader is deferred to (Bullo and Murray, 1999). An error function is a map $\phi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, such that $\phi(x,r) \geq 0$ and $\phi(x,r) = 0$ iff $x = r$. We shall also impose that $\mathrm{d}_2\phi(x,r) = -\mathrm{d}_1\phi(x,r)$ where d_1 is the exterior derivative with respect to x and d_2 the exterior derivative with respect to r. This will allow the time derivative of $\phi(x,r)$ being expressed by the familiar expression:

$$\begin{aligned} \frac{d}{dt}\phi(x,r) &= \mathrm{d}_1\phi(x,r).\dot{x} + \mathrm{d}_2\phi(x,r).\dot{r} \\ &= \mathrm{d}_1\phi(x,r)(\dot{x} - \dot{r}) \end{aligned} \qquad (5)$$

We shall say that the error function is (uniformly) quadratic lower bounded if there is a scalar $b \geq 0$ such that:

$$\phi(x,r) \geq b\|\mathrm{d}_1\phi(x,r)\|^2 \qquad (6)$$

Note than in abstract manifolds this condition may only hold locally according to the topology of the manifold.

3. TRACKING FOR NONHOLONOMIC SYSTEMS

3.1 *Problem formulation*

The goal of this paper is to describe an algorithm to track a desired position reference $r(t)$ disregarding the rigid body orientation. A control law $u(g)$ solves the position tracking problem if:

- The tracking error $\phi(t) = \phi(x(t), r(t))$ and the controls u_i are bounded for all time.
- The tracking error asymptotically decays to zero, $\lim_{t\to\infty} \phi(t) = 0$

It is usual to include another requirement when only feasible trajectories are being tracked, namely that $\phi(0) = 0 \Rightarrow \phi(t) = 0$. However this requirement may not be satisfied if one wishes to track trajectories not feasible by all the states of the system. Suppose that $\phi(0) = 0$ and that $\frac{d}{dt}r(t) \notin Span\{X_1, X_2, X_3, X_4\}$ under this scenario one can never guarantee that the error function will remain zero.

3.2 *Regularity and boundedness assumptions*

We shall assume the following regularity and boundedness properties:

- $\phi(x, r) \in C^2$.
- $r(t)$ is twice differentiable.
- $\sup_{t\in\mathbb{R}} \|r(t)\| < \infty$, $\sup_{t\in\mathbb{R}} \|\dot{r}(t)\| < \infty$ and $\sup_{t\in\mathbb{R}} \|\ddot{r}(t)\| < \infty$.

We assume that the reference trajectory is twice differentiable which is not a restrictive assumption since it is desirable for the reference to be as smooth as possible. Boundedness assumptions on the reference trajectory are also standard assumptions.

3.3 *Intuitive motivation*

To achieve exponential tracking of the rigid body position it would desirable that the vector field X describing the motion of the rigid body could be chosen to be $X = \dot{r} - \lambda(\mathrm{d}_1\phi(x, r))^T$. We use the metric $(g_{ij} = \beta \mathbf{I}_{3\times 3})$ on $\mathbb{R}^3$ to transform the covector $\mathrm{d}_1\phi(x, r)$ in the vector $g^{ij}(\mathrm{d}_1\phi(x, r))_j = \frac{1}{\beta}(\mathrm{d}_1\phi(x, r))^i = \frac{1}{\beta}(\mathrm{d}_1\phi(x, r))^T = \lambda(\mathrm{d}_1\phi(x, r))^T$. Therefore, by using $V_1 = \phi(x, r)$ as a candidate Lyapunov function one immediately sees that:

$$\begin{aligned}\frac{d}{dt}V_1 &= \mathrm{d}_1\phi(x, r).(\dot{x} - \dot{r}) \\ &= \mathrm{d}_1\phi(x, r).\big(\dot{r} - \lambda(\mathrm{d}_1\phi(x, r))^T - \dot{r}\big) \\ &= -\lambda \mathrm{d}_1\phi(x, r).(\mathrm{d}_1\phi(x, r))^T \qquad (7)\end{aligned}$$

which is negative semi-definite, negative definiteness is a consequence of the quadratic nature of ϕ. In fact, using the inequality in (6):

$$\begin{aligned}\frac{d}{dt}V_1 &= -\lambda \mathrm{d}_1\phi(x, r).(\mathrm{d}_1\phi(x, r))^T \\ &= -\lambda\|\mathrm{d}_1\phi(x, r)\|^2 \le -\frac{\lambda}{b}\phi(x, r) \qquad (8)\end{aligned}$$

It is not always possible to freely assign the vector field $\dot{x}$ due to the kinematic restrictions of the system. However the above observation suggests the following approach to solve the problem:

- Use roll, pitch and yaw inputs to align the vector field $X_r = \begin{bmatrix}\mathbf{0}_{3\times 3} & X \\ \mathbf{0}_{1\times 3} & 0\end{bmatrix}$ with X_4.
- Project X_r on X_4, to determine the forward velocity control input.

This approach will now be described in more detail.

3.4 *Orientation control*

To ensure that X_r belongs to $Span\{X_1, ..., X_4\}$ one must derive a control law that stabilizes the system in the following set $\Psi = \{g \in SE(3) \, : < X_r, X_5(g) >_g = 0, < X_r, X_6(g) >_g = 0\}$. We can build a candidate Lyapunov function measuring the "distance" to the set Ψ. Let $\psi_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \quad i = 1, 2$ be two error functions and consider the following Lyapunov candidate function:

$$V_2 = \psi_1(< X_r, X_5 >, 0) + \psi_2(< X_r, X_6 >, 0) \qquad (9)$$

After some tedious algebra (Tabuada and Lima, 2000), it can be shown that its time derivative is given by:

$$\begin{aligned}\frac{d}{dt}V_2 = \mathrm{d}\psi_1\big(&< \{0, \ddot{r} - \lambda\frac{d}{dt}(\mathrm{d}_1\phi(x, r)|_{xfixed})^T\}, X_5 > \\ &-\lambda u_4 < \frac{\partial^2\phi(x, r)}{\partial x^i x^j}X_4, X_5 > \\ &+ u_1 < X_r, X_6 > - u_3 < X_r, X_4 >\big) \\ + \mathrm{d}\psi_2\big(&< \{0, \ddot{r} - \lambda\frac{d}{dt}(\mathrm{d}_1\phi(x, r)|_{xfixed})^T\}, X_6 > \\ &-\lambda u_4 < \frac{\partial^2\phi(x, r)}{\partial x^i x^j}X_4, X_6 > \\ &+ u_2 < X_r, X_4 > - u_1 < X_r, X_5 >\big) \qquad (10)\end{aligned}$$

This means that if we choose $\phi(x, r)$ in such a way that $< \frac{\partial^2\phi(x,r)}{\partial x^i x^j}X_4, X_5 >= 0$ and $< \frac{\partial^2\phi(x,r)}{\partial x^i x^j}X_4, X_6 >= 0$ and as long as $< X_r, X_4 > \neq 0$ we can use u_2 and u_3 to exponentially steer the rigid body towards the set Ψ. Before stating this result we will give a more useful characterization of the allowed error functions:

Proposition 3.1. The requirement $< \frac{\partial^2\phi(x,r)}{\partial x^i x^j}X_4, X_5 >= 0 =< \frac{\partial^2\phi(x,r)}{\partial x^i x^j}X_4, X_6 >$ is satisfied iff $\phi(x, r) = \frac{1}{2}k(r)(x - r)^T(x - r)$, where $k(r)$ is a smooth function of r.

Now we are ready to state the following result.

Proposition 3.2. (Exponential stabilization in Ψ). For all initial conditions in the open and dense set $\Sigma = \{g \in SE(3) \, : < X_r, X_4(g) >_g \neq$

$0\}$ and all the error functions of the form $\phi(x,r) = \frac{1}{2}k(r)(x-r)^T(x-r)$ the control law:

$$
\begin{aligned}
u_1 &= 0 \qquad (11)\\
u_2 &= -\frac{\rho_2 \mathrm{d}\psi_2}{< X_r, X_4 >} \\
&\quad - \frac{< \{0, \ddot{r} - \lambda \frac{d}{dt}(\mathrm{d}_1\phi(x,r)|_{x fixed})^T\}, X_6 >}{< X_r, X_4 >} \\
u_3 &= \frac{\rho_3 \mathrm{d}\psi_1}{< X_r, X_4 >} \\
&\quad + \frac{< \{0, \ddot{r} - \lambda \frac{d}{dt}(\mathrm{d}_1\phi(x,r)|_{x fixed})^T\}, X_5 >}{< X_r, X_4 >}
\end{aligned}
$$

for $\rho_2,\ \rho_3 > 0$ exponentially stabilizes the system (1) in the set Ψ.

Proof: Consider the Lyapunov candidate function (9) and its time derivative, given by (10). Substituting the control law (11) and taking into account the special form of the error function, one gets:

$$\frac{d}{dt}V_2 = -\rho_3(\mathrm{d}\psi_1)^2 - \rho_2(\mathrm{d}\psi_2)^2 \qquad (12)$$

which is negative semidefinite. Negative definiteness is proved with an argument similar to the proof of (7), let b^{ψ_1}, b^{ψ_2} be the quadratic lower bounding constants for the functions ψ_1, ψ_2 as defined in (6), respectively. It follows that:

$$\frac{d}{dt}V_2 \leq -\frac{\rho_3}{b^{\psi_1}}\psi_1 - \frac{\rho_2}{b^{\psi_2}}\psi_2 \qquad (13)$$

To show that trajectories never leave the set Σ it is enough to consider that $\dot{V}_2 \leq 0$, therefore the projection of X_r over X_4 never decreases and thus can never be zero.

Remarks: The special form of the error function is not necessary to stabilize the system in the set Ψ, however it is very useful since it decouples the orientation control from the position control. It will allow us to chose a control law for u_4 in the next section without disturbing the orientation kinematics. However it reduces the set of possible error functions, forbidding the use of different weights for the error along different directions (one is forced to use $k(r)$ in all directions). This can also be seen as a direct consequence of the reduced set of metrics compatible with the kinematics connection.

In control law (11) u_1 was chosen to be zero, implying that it is not necessary that the rigid body possesses roll control to stabilize it in Ψ. In fact, similar control laws could be developed by choosing u_2 or u_3 to be zero. Roll control is still important if pitch or yaw control fails, constituting a useful redundancy. What is more useful in certain situations is to be able to chose which actuators to use for optimizing fuel consumption or other optimality criteria during the mission, however this approach will not be further addressed in this paper.

Note that control law (11) uses the acceleration of the reference trajectory which is not usual in trajectory tracking. This can be easily explained if one realizes that the attitude control is tracking velocities in trying to align X_r with X_4, therefore since (11) can be viewed as a generalized PD controller it needs acceleration information to accomplish this goal.

Unfortunately control law 11 does not guarantees convergence for all initial conditions, but only for an open and dense set of $SE(3)$. Nevertheless this is the best that can be achieved since $SE(3)$ is not a simply connected space.

3.5 *Position Control*

Since the orientation of the rigid body is converging to the set Ψ by the action of control inputs u_2 and u_3, it remains to control the forward velocity through control input u_4. The control law for u_4 should be proportional to a measure of the alignment between X_r and X_4, this can trivially be achieved by projecting the reference vector field X_r on X_4, resulting in:

$$
\begin{aligned}
u_4 &= \frac{< X_r, X_4 >}{< X_4, X_4 >} = \frac{1}{\beta} < X_r, X_4 > \\
&= \lambda < X_r, X_4 > \qquad (14)
\end{aligned}
$$

Combining (14) with (11) we can asymptotically track the desired reference. This constitutes the main contribution of the paper and is expressed in the following:

Theorem 3.3. (Asymptotical position tracking). For all initial conditions in the set Σ and all error functions of the form $\phi(x,r) = \frac{1}{2}k(r)(x-r)^T(x-r)$, control law (11) and (14) makes the system (1) asymptotically track the desired reference $r(t)$.

In order to prove the result we will need the following standard lemma whose proof can be found in (Khalil, 1996) Appendix A.2.

Lemma 3.4. Let $f(x) : D \to \mathbb{R}^n$, $D \subset \mathbb{R}^n$ be a locally Lipschitz vector field on D. If the solution $x(t)$ is bounded and belongs to D for $t \geq 0$, then its positive limit set L^+ is a nonempty, compact, invariant set. Moreover, $x(t) \to L^+$ as $t \to \infty$.

Due to space limitations we present only a sketch of the proof:

Proof: This proof sketch will consist of the following steps: boundedness of system trajectories, convergence of trajectories to the largest invariant set in Ψ and equality between largest invariant set in Ψ and desired reference $r(t)$.

Boundedness of trajectories. The trajectories of the rotation matrices (living in $SO(3)$) are bounded since $SO(3)$ is a compact space. We only need to show that position of the rigid body is also bounded. By using the fact that the trajectories of the system $\dot{x}' = X(x') = \dot{r} - \mathrm{d}_1\phi(x', r)$ are bounded since $\frac{d}{dt}\phi \leq 0$ as shown in (7) it can be shown that trajectories of (1) with control law (14) and (11) are also bounded.

Convergence to the largest invariant set in Ψ. The system (1) with control laws (11) and (14) is locally Lipschitz since the boundedness assumptions (3.2) on $\phi(x, r)$ and $r(t)$ easily imply that $\frac{\partial \dot{g}}{\partial g}$ is continuous on Σ. Therefore on any compact neighborhood the derivative of $\dot{g}$ with respect to g is bounded, implying local Lipschitz continuity. By applying Lemma 3.4 we conclude that the positive limit set is an invariant set. From (9) we know that trajectories approach Ψ asymptotically, therefore by Lemma 9 they approach the largest invariant set contained in Ψ.

The largest invariant set in Ψ. To study the largest invariant set in Ψ we start by noting that $g \in \Psi \Rightarrow X_4 = \eta X_r$ for a scalar η and the position kinematics is simplified to $\dot{x} = \dot{r} - \lambda(\mathrm{d}_1\phi(x, r))^T$. Therefore the largest invariant set in Ψ is the desired reference $r(t)$ as shown in (7).

4. SIMULATION RESULTS

In this section some simulations results are presented for the $SE(3)$ and the $SE(2)$ case. For the $SE(3)$ the used error functions and gains were $\phi(x, r) = \frac{1}{2}(x - r)^T(x - r)$, $\psi_1(a, b) = \psi_2(a, b) = \frac{1}{2}(a - b)^2$, $\rho_1 = 10$ and $\rho_2 = 10$. The metric scalar β was chosen to be unitary. With these values the desired reference was an helix given by $r(t) = \left(\sin(\frac{t}{10}), \cos(\frac{t}{10}), \frac{t}{10}\right)$, $t \in [0, 100]$. The errors between the desired trajectory $r(t)$ and the real trajectory $x(t)$ are represented in Figure 1 for an initial position of $x(0) = (-10, -30, -5)$ and an initial orientation of $R = \mathbf{I}_{3\times3}$. Convergence is very fast and the reference trajectory is tracked with good precision. This motivates the use of more challenging references such as:

$$r(t) = \begin{cases} (t, t) & 0 < t \leq 30 \\ (60 - t, t) & 31 < t \leq 60 \\ (-60 + t, t) & 61 < t \leq 100 \end{cases} \tag{15}$$

for the $SE(2)$ case. Note that the reference is not twice differentiable violating the conditions of Theorem 3.3. This implies that the system will lose track of the reference at the points of non-differentiability as can be seen in Figure 2. Even in this case the results are very impressive since the trajectory is retracked very quickly after being lost. To turn the simulations more realistic the same trajectory was simulated with bounded actuators by restricting the linear and angular velocities were restricted to the set $[-5, 5]$. The results are depicted in Figure 3. The convergence

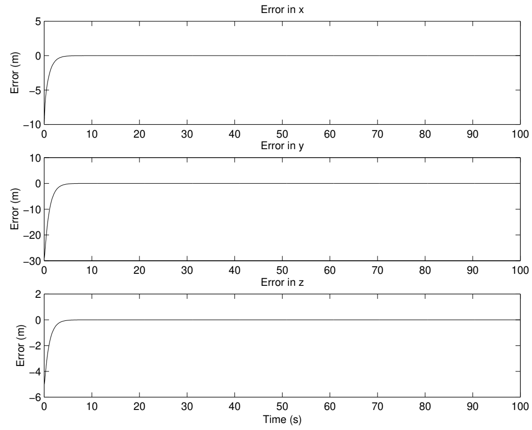

Fig. 1. Tracking error $(r(t) - x(t))$ for the $SE(3)$ case, initial position $x(0) = (-10, -30, -5)$, initial orientation $R = \mathbf{I}_{3\times3}$.

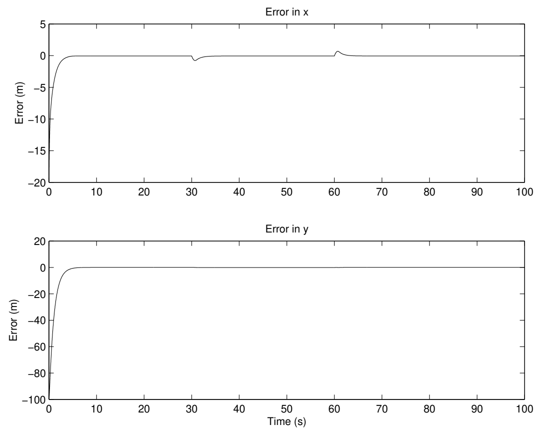

Fig. 2. Tracking error $(r(t) - x(t))$ for the $SE(2)$ case, initial position $x(0) = (20, 100)$, initial orientation $R = -\mathbf{I}_{2\times2}$.

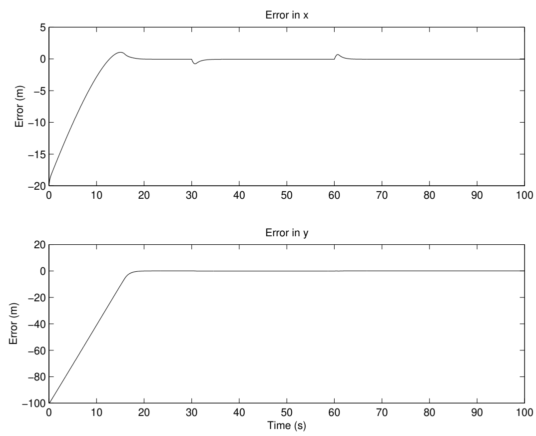

Fig. 3. Tracking error $(r(t) - x(t))$, for the $SE(2)$ case with bounded actuators, initial position $x(0) = (20, 100)$, initial orientation $R = -\mathbf{I}_{2\times2}$.

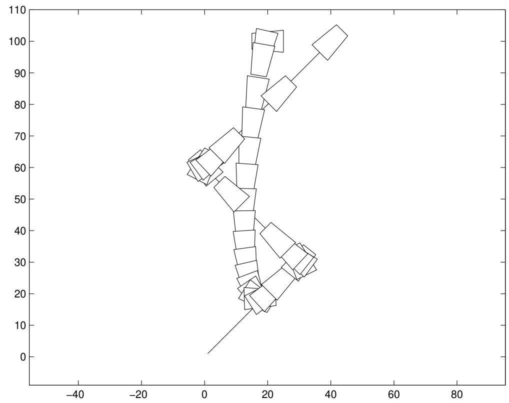

Fig. 4. Reference trajectory and vehicle location for the $SE(2)$ case with bounded actuators, initial position $x(0) = (20, 100)$, initial orientation $R = -\mathbf{I}_{2\times2}$.

time is much greater and the initial part of the trajectory is not tracked at all as can be seen from Figure 4. This was expected since the initial condition is far from the trajectory, however at the points of non-differentiability of the reference the results are very similar with the unrestricted actuators case, evidencing the good performance and robustness for situations not explicitly taken in to account in the theoretical development.

5. CONCLUSIONS

In this paper we have studied the problem of tracking a desired position for an underactuated rigid body in the special euclidean group. It is was shown that the problem is solvable using static state feedback on a open and dense subset of $SE(3)$ even if roll control is not possible. The development of the control law was done in a coordinate free way thereby avoiding the unnecessary complications often imposed by possible parameterizations of $SE(3)$. The need to decouple the position motion from the orientation motion led to a reduction of the set of functions measuring the error between the rigid body desired and actual position. This is a direct consequence of the also reduced set of left invariant metrics compatible with the kinematic connection on $SE(3)$. Asymptotic convergence towards the reference trajectory was shown and several simulations were included to shown the algorithm good performance even with non-differentiable reference trajectories.

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